Arc | AIEEE

AIEEE
Mock Test – I
Answers and Explanations

Mathematics			
1	b	21	b
2	a	22	a
3	b	23	d
4	d	24	d
5	d	25	c
6	c	26	c
7	d	27	b
8	c	28	b
9	c	29	c
10	a	30	c
11	d	31	b
12	a	32	c
13	c	33	b
14	d	34	d
15	c	35	b
16	a		
17	b		
18	b		
19	c		
20	c		

Mathematics

1. b $\sin^{-1}x + \sin^{-1}\left(\frac{1}{x}\right) + \cos^{-1}x + \cos^{-1}\left(\frac{1}{x}\right)$

$= \sin^{-1}x + \cos^{-1}x + \sin^{-1}\left(\frac{1}{x}\right) + \cos^{-1}\left(\frac{1}{x}\right)$

$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$

2. a Putting $2x + y = t$,

$2 + \frac{dy}{dx} = \frac{dt}{dx}$

The given equation can be written

$(2t-2) - (t+1)\left(\frac{dt}{dx} - 2\right) = 0$

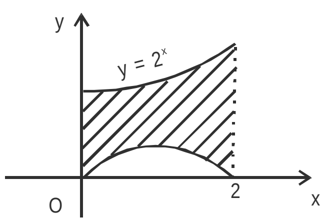
$\Rightarrow \frac{dt}{dx} = \frac{4t}{t+1} \Rightarrow \frac{1}{4}\int\left(1 + \frac{1}{t}\right)dt = x + c$

3. b The given integral can be written as

$\int_{e^{-1}}^{1} -\frac{\ell n x}{x}dx + \int_{1}^{e^2}\frac{\ell n x}{x}dx$

$= -\left[\frac{(\ell n x)^2}{2}\right]_{e^{-1}}^{1} + \left[\frac{(\ell n x)^2}{2}\right]_{1}^{e^2} = \frac{5}{2}$

4. d



Area $= \int_0^2\left(2^x - \left(2x - x^2\right)\right)dx$

$= \frac{3}{\log_e 2} - \frac{4}{3} = 3\log_2 e - \frac{4}{3}$

5. d $I = \int\left(\left(x^2\right)^3\left(1 + x + x^4\right)^3 x\left(2 + 3x + 6x^4\right)\right)dx$

$= \int\left(x^2 + x^3 + x^6\right)^3\left(2x + 3x^2 + 6x^5\right)dx$

Let $x^2 + x^3 + x^6 = t$

$(2x + 3x^2 + 6x^5)\,dx = dt$

$I = \int t^3\,dt = \frac{t^4}{4} + c$

$= \frac{\left(x^2 + x^3 + x^6\right)^4}{4} + c = \frac{x^8\left(1 + x + x^4\right)^4}{4} + c$

6. c $abc\begin{vmatrix} bc & 1 & ab+ac \\ ca & 1 & bc+ab \\ ab & 1 & ac+bc \end{vmatrix}$

$C_3 \to C_3 + C_1$

$= abc\begin{vmatrix} bc & 1 & ab+bc+ca \\ ca & 1 & ab+bc+ca \\ ab & 1 & ab+bc+ca \end{vmatrix}$

$= abc\ (ab + bc + ca)\begin{vmatrix} bc & 1 & 1 \\ ca & 1 & 1 \\ ab & 1 & 1 \end{vmatrix}$

$= 0$

7. d $y' = 3ax^2 + 2bx + c \geq 0$ for all x.

$\Rightarrow a > 0$ and $D \leq 0$

$\Rightarrow a > 0$ and $4b^2 - 12ac \leq 0$

$\Rightarrow a > 0$ and $b^2 - 3ac \leq 0$

8. c $\lim\limits_{x\to 0^+}\frac{e^{-1/x} - e^{1/x}}{e^{-1/x} + e^{1/x}}$

$= \lim\limits_{x\to 0^+}\frac{e^{-2/x} - 1}{e^{-2/x} + 1} = \frac{0-1}{0+1} = -1$

9. c $\cos A + \cos C = 2(1 - \cos B)$

$\Rightarrow 2\cos\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right) = 2 \times 2\sin^2\frac{B}{2}$

$\Rightarrow \cos\frac{A-C}{2} = 2\sin\frac{B}{2}$

$$\Rightarrow 2\cos\left(\frac{A-C}{2}\right)\cos\frac{B}{2} = 4\sin\frac{B}{2}\cos\frac{B}{2}$$

$$\Rightarrow 2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right) = 2\sin B$$

$$\Rightarrow \sin A + \sin C = 2\sin B$$

$$\Rightarrow a + c = 2b$$

10. a Let A be the event:
'Maximum number is not more than 10'
B: 'Minimum number is not less than 5'.

then $P(B/A) = \frac{{}^{6}C_2}{{}^{10}C_2} = \frac{1}{3}$

11. d We know $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
But it is given that
$P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$
$\therefore P(A \cap B) = P(A)\ .\ P(B)$
This shows that A and B are independent events.

Hence, $P\left(\frac{A}{B}\right) = P(A)$

12. a $f'(x) = 2x - 2 = 2(x-1) < 0$ if x < 1, i.e. $x \in (-\infty, 1)$.
Hence, f is strictly decreasing in $(-\infty, 1)$.

13. c Let the polygon has n sides.
$\therefore$ Number of diagonals = ${}^{n}C_2 - n = 35$

$$\Rightarrow \frac{(n-1)n}{2} - n = 35$$

$\Rightarrow n^2 - n - 2n = 70$
$\Rightarrow n^2 - 3n - 70 = 0$
$\Rightarrow (n - 10)(n + 7) = 0$
$\Rightarrow$ n = 10 or –7
Hence, number of sides = Number of vertices = 10

14. d

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

$\Rightarrow a(cb - a^2) - b(b^2 - ca) + c(ba - c^2) = 0$
$\Rightarrow a^3 + b^3 + c^3 - 3abc = 0$
$\Rightarrow (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$
$\Rightarrow a + b + c = 0$
or $a^2 + b^2 + c^2 = ab + bc + ca$

15. c Given parabola is $y^2 - kx + 8 = 0$ or $y^2 = k\left(x - \frac{8}{k}\right)$

So vertex of the parabola is $\left(\frac{8}{k}, 0\right)$ and length of latus rectum is k,
i.e. $4a = k \Rightarrow a = \frac{k}{4}$

Hence, the equation of directrix is $x = \frac{8}{k} - a$

$$\Rightarrow x = \frac{8}{k} - \frac{k}{4}$$

But the directrix is given to be x = 1.

Therefore, we must have $\frac{8}{k} - \frac{k}{4} = 1 \Rightarrow k = 4, -8$

16. a The given circles are

$x^2 + y^2 = 1$ ...(i)

$x^2 + y^2 + 10y + 24 = 0$ ...(ii)

$x^2 + y^2 - 8x + 15 = 0$ ...(iii)

Equation of radical axis of (i) and (ii) is

$$10y + 25 = 0 \Rightarrow y = -\frac{5}{2}$$

Equation of radical axis of (ii) and (iii) is
$8x + 10y + 9 = 0$
$\Rightarrow 8x - 25 + 9 = 0 \Rightarrow x = 2$

$\therefore$ Radical centre $\left(2, -\frac{5}{2}\right)$

17. b

$$\frac{d^3y}{dx^3} = 0 \Rightarrow \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = 0 \Rightarrow \frac{d^2y}{dx^2} = A$$

$$\Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = A \Rightarrow \frac{dy}{dx} = Ax + B \Rightarrow y = \frac{Ax^2}{2} + Bx + C$$

$$\Rightarrow y = ax^2 + bx + c;\ a = \frac{A}{2}, b = B, c = C$$

18. b The required condition is

$$\frac{k+1}{k} = \frac{8}{k+3} = \frac{4k}{3k-1}$$

$\Rightarrow k^2 - 4k + 3 = 0$ and $k^2 - 3k + 2 = 0$
$\Rightarrow$ k = 1, 3 and 1, 2
$\therefore$ One common value.

19. c $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$BA = \begin{bmatrix} 0 & 0 \\ 20 & 0 \end{bmatrix}$

20. c $\alpha + \beta = 4$ ... (i)

$\alpha\beta = a$ ... (ii)

$\gamma + \delta = 36$ ... (iii)

$\gamma\delta = b$ ... (iv)

Now α, β, γ and δ are k, kr, kr^2 and kr^3 respectively, since they are in GP.

(i) $\Rightarrow$ k(1 + r) = 4

(iii) $\Rightarrow kr^2(1 + r) = 36$

(iii) ÷ (i) $\Rightarrow$ $r^2 = 9 \Rightarrow r = +3$

Since it is an increasing GP.

Now $\alpha = 1$, $\beta = 3$, $\gamma = 9$, $\delta = 27$

And $\alpha\beta = 3$, $\gamma\delta = 243$

So, a = 3, b = 243

21. b Let $Y = \frac{aX + b}{c}$. Then $\overline{Y} = \frac{1}{c}\left(a\overline{X} + b\right)$

$\Rightarrow Y - \overline{Y} = \frac{a}{c}\left(X - \overline{X}\right)$

$\Rightarrow \frac{1}{N}\Sigma(Y - \overline{Y})^2 = \frac{a^2}{c^2}\frac{1}{N}\Sigma\left(X - \overline{X}\right)^2$

$\therefore$ Standard deviation of Y = $\sqrt{\frac{a^2}{c^2}\frac{1}{N}\Sigma\,(X - \overline{X})^2}$

$= \sqrt{\frac{a^2}{c^2}\sigma^2} = \left|\frac{a}{c}\right|\sigma$

22. a Use options.

23. d $a.p = \frac{a \cdot (b \times c)}{[abc]} = \frac{[abc]}{[abc]} = 1$

$b \cdot p = 0, b \cdot q = 1, c \cdot q = 0, c \cdot r = 1, a \cdot r = 0$

$\Rightarrow 1 + 0 + 1 + 0 + 1 + 0 = 3$

24. d The given point satisfies the given equation of straight line. So the length of perpendicular is zero.

25. c $\int (3x^3 - 17)e^{2x}dx = \left(Ax^3 + Bx^2 + Dx + E\right)e^{2x} + C$

Differentiating both the sides, we obtain

$(3x^3 - 17)e^{2x} = 2\left(Ax^3 + Bx^2 + Dx + E\right)e^{2x}$

$+ e^{2x}\left(3Ax^2 + 2Bx + D\right)$

i.e. $3x^3 - 17 = 2Ax^3 + (2B + 3A)x^2 + (2D + 2B)x + (2E + D)$

Equating the coefficients at the equal powers of x in the left and right sides of the identity, we get

$2A = 3; 2B + 3A = 0; 2D + 2B = 0; 2E + D = -17$

Solving the system, we obtain

$A = \frac{3}{2}; B = \frac{-9}{4}; D = \frac{9}{4}; E = \frac{-77}{8}$

Hence, $I = \left(\frac{3}{2}x^3 - \frac{9}{4}x^2 + \frac{9}{4}x - \frac{77}{8}\right)e^{2x} + C$

26. c Here |z – zi| = 1

$\Rightarrow$ |x + iy – xi + y| = 1

$\Rightarrow$ |(x + y) + i(y – x)| = 1

$\Rightarrow \sqrt{(x + y)^2 + (y - x)^2} = 1$

$\Rightarrow 2(x^2 + y^2) = 1$

$\Rightarrow x^2 + y^2 = \frac{1}{2}$

Hence, z lies on a circle.

Alternative method:
or |z| |1 – i| = 1

$|z| \times \sqrt{2} = 1$

$|z| = \frac{1}{\sqrt{2}}$

27. b We must have

$2 - 2x - x^2 \geq 0$

$\Rightarrow x^2 + 2x - 2 \leq 0$

$\Rightarrow (x + 1)^2 - 3 \leq 0$

$\Rightarrow -\sqrt{3} \leq (x + 1) \leq \sqrt{3}$

$-1 - \sqrt{3} \leq x \leq -1 + \sqrt{3}$

28. b Let f(x) = π
$\therefore$ f(–x) = π as f(x) is constant function.
$\therefore$ f(x) = f(–x) $\Rightarrow$ even function.
Again f(x + T) = π
and f(x) = π
$\Rightarrow$ f(x + T) = f(x)
$\Rightarrow$ f(x) is periodic, but we cannot find the fundamental period.

29. c $(1 + \tan x)(1 + \tan y) = (1 + \tan x)\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)$

$= (1+\tan x)\left(1+\frac{1-\tan x}{1+\tan x}\right) = 2$

30. c $\frac{11+13+15+\text{........to n terms}}{4+7+10+\text{........to 5 terms}} = 1.5$

$\Rightarrow \frac{\frac{n}{2}[11\times 2+(n-1)2]}{\frac{5}{2}[2\times 4+4\times 3]} = 1.5$

$\Rightarrow n(10+n) = 1.5\times 2.5(20)$

$\Rightarrow n^2 + 10 \quad n - 75 = 0$

$\Rightarrow n = \frac{-10\pm\sqrt{100+300}}{2} = \frac{-10\pm 20}{2} = -15, 5$

n = – 15 is not possible, i.e. n = 5

31. b $\sum_{r=1}^{4n+11} i^r = i + i^2 + i^3 + \sum_{r=4}^{4n+11} i^r = i - 1 - i + 0$
= –1

32. c If a, b, c are in GP, then a + b, b + b, c + b are in HP.

$\Rightarrow (2b) = \frac{2(a+b)(b+c)}{(a+b)+(c+b)}$

$\Rightarrow$ b(a + 2b + c) = (a + b) (b + c)
$\Rightarrow$ b(a + c) + $2b^2$ = ab + ac + b^2 + ac
$\Rightarrow$ b^2 = ac ($\because$ a, b, c are in GP)

33. b Let f(x) be periodic with period $\lambda, \lambda \neq 0, \lambda > 0$

$\therefore f(x+\lambda) = f(x)$

$\Rightarrow \cos(\cos(x+\lambda)) + \cos(\sin(x+\lambda)) + \sin(4(x+\lambda))$
= cos(cosx) + cos(sinx) + sin4x

Put x = 0
$\cos(\cos\lambda) + \cos(\sin\lambda) + \sin(4\lambda)$
= cos(1) + cos(0) + sin0
$= \cos(\sin\pi/2) + \cos(\cos\pi/2) + \sin(2\pi)$
$\Rightarrow \lambda = \pi/2$

34. d Let l, m, n be the DC's of the line of the common perpendicular (or SD) to the two given lines.
Then, we have
–4l + 3m + 2n = 0
and –4l + m + n = 0
Solving these, we get

$\frac{l}{3-2} = \frac{m}{-8+4} = \frac{n}{-4+12}$

or $\frac{l}{1} = \frac{m}{-4} = \frac{n}{8} = \frac{\sqrt{(l^2+m^2+n^2)}}{\sqrt{(1)^2+(-4)^2+(8)^2}} = \frac{1}{9}$

$\therefore$ DC's of SD are $\frac{1}{9}, \frac{4}{-9}, \frac{8}{9}$

Also, A(–3, 6, 0) is a point on first line and B(–2, 0, 7) is a point on second line, then

$SD = \left|(-2+3)\frac{1}{9} + (0-6)\left(-\frac{4}{9}\right) + (7-0)\left(\frac{8}{9}\right)\right|$

= 9

35. b For mutually exclusive events
$P(A\cup B\cup C) = P(A)+P(B)+P(C)$

$= \frac{2}{3}+\frac{1}{4}+\frac{1}{6}$

$= \frac{13}{12} > 1$

Which is not possible.